17. State and prove Laurent's theorem.

18. Prove that All conformal mappings which map the unit disk Δ onto itself and the point $z_0, |z_0| < 1$, onto 0 must be of the form $\phi(z) = e^{i\theta}\left(\frac{z - z_0}{1 - z_0 z}\right)$ for some $\theta \in \mathbb{R}$.

19. State and prove Borel Caratheodary theorem.

20. State and prove Riemann's Removable Singularity theorem.

APRIL/MAY 2024

DMA31/GMA31 — COMPLEX ANALYSIS – I

Time : Three hours Maximum : 75 marks

Shanmuga Industries Arts & Science College T.V.Malai

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. When a function f is said to be differentiable?

2. Define binomial series.

3. Define winding number of a curve.

4. State Taylor's theorem.

5. Write any two basic properties of mobius transformation.

6. Define loxodromic mobius transformation

7. Define local maximum.

8. State Schwarz' lemma.

9. Define singular point.

10. Define meromorphic function.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) State and prove chain rule for differentiation of composite functions.

Or

(b) State and prove the uniqueness theorem for power series.

12. (a) State and prove Cauchy integral formula.

Or

(b) Prove that every zero of an analytic function $f\ (\not\equiv 0)$ is isolate.

13. (a) Given three distinct points z_1, z_2, z_3 in C_∞, prove that there exists a unique mobius transformation $T(z)$ such that $T(z_1)=0$, $T(z_2)=1$ and $T(z_3)=\infty$.

Or

(b) Prove that the four distinct points z, z_1, z_2, z_3 in C_∞ all lie on a circle or on a line iff their cross - ratio (z, z_1, z_2, z_3) is a real number.

14. (a) State and prove Hadamard's three lines theorem.

Or

(b) State and prove Gauss's theorem.

15. (a) Let N be a deleted neighborhood of z_0 such that f is analytic in N. If f has an isolated singularity at z_0, the prove that z_0 is a pole of order n iff there are positive constants C_1 and C_2 such that $C_2 \le |(z-z_0)^n f(z)| \le C_1$ f or some deleted neighborhood N_0 of z_0 such that $N_0 \subset N$.

Or

(b) Discuss the singularities of $f(z) = \dfrac{z-1-i}{z^2-(4+3i)z+1(1+5i)}$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let $f:\Omega \to C$, where Ω is an open set in C containing the point z_0 and $f(z)=u(x,y)+iv(x,y)$ for $z=x+iy \in \Omega$. Suppose that.

(a) u_x, u_y, v_x and v_y exist at each point in a neighborhood of z_0 and are continuous at z_0.

(b) the C–R equations are valid at z_0. Then prove that f is differentiable at z_0 and $f'(z_0)=u_x(z_0)+iv_x(z_0)$.